Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in
Core Mathematics 4 (6666/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2015
Publications Code UA041202

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\lfloor$ The second mark is dependent on gaining the first mark
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme		Marks
1. (a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$ $=2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$	See notes below!	
	$=2+\frac{5}{4}x;-\frac{25}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\left\{x=\frac{1}{10}\Rightarrow(4+5(0.1))^{\frac{1}{2}}=\sqrt{4.5}=\sqrt{\frac{9}{2}}=\underline{\underline{\frac{3}{\sqrt{2}}}}=\frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$		
	$=\frac{3}{2}\sqrt{2}$	$\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
			[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}=2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$	See notes	M1
	So, $\frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}=\frac{543}{256}$		
	yields, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	$\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ **or** $\frac{256}{181}$ etc.	A1 oe
			[2] **8**

	Question 1 Notes	
1. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion.
	M1	Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and **where** $k\neq1$.
	A1	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx).
	Note	(kx), $k\neq1$, must be consistent (on the RHS, not necessarily on the LHS) in a candidate's expansion.

1. (a) ctd.	**Note**	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(5x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+...\right]$ because (kx) is not consistent.
	Note	**<u>Incorrect bracketing:</u>** $2\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x^2}{4}\right)+...\right]$ is B1M1A0 unless recovered.
	A1	$2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1	Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	SC	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 then **allow Special Case 2nd A1 for either** **SC:** $2\left[1+\frac{5}{8}x;...\right]$ or **SC:** $2\left[1+...-\frac{25}{128}x^2+...\right]$ or **SC:** $\lambda\left[1+\frac{5}{8}x-\frac{25}{128}x^2+...\right]$ or **SC:** $\left[\lambda+\frac{5\lambda}{8}x-\frac{25\lambda}{128}x^2+...\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, **OR SC:** for $2+\frac{10}{8}x-\frac{50}{128}x^2+...$ (i.e. for not simplifying their correct coefficients.)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{5x}{4}\right)^2+...\right]$, where $k=-\frac{5}{4}$ and not $\frac{5}{4}$ and achieve $2-\frac{5}{4}x-\frac{25}{64}x^2+...$ will get B1M1A1A0A1
	Note	Ignore extra terms beyond the term in x^2.
	Note	You can ignore subsequent working following a correct answer.
(b)	**B1**	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e. (Ignore how $k=\frac{3}{2}$ is found.)
(c)	**M1**	Substitutes $x=\frac{1}{10}$ or 0.1 into their binomial expansion found in part (a) which must contain both an x term and an x^2 term (or even an x^3 term) **and** equates this to either $\frac{3}{\sqrt{2}}$ or their $k\sqrt{2}$ from (b), where k is a numerical value.
	Note	M1 can be implied by $\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\underline{\underline{\frac{3}{\sqrt{2}}}}$ = awrt 2.121
	Note	M1 ***can be implied*** by $\frac{1}{k}\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	Note	M1 ***cannot be implied*** by $(k)\left(\text{their } \frac{543}{256}\right)$, with their k found in part (b).
	A1	$\frac{181}{128}$ **or any equivalent fraction**, eg: $\frac{362}{256}$ or $\frac{543}{384}$. Also allow $\frac{256}{181}$ **or any equivalent fraction.**
	Note	Also allow A1 for $p=181, q=128$ or $p=181\lambda, q=128\lambda$ or $p=256, q=181$ or $p=256\lambda, q=181\lambda$, where $\lambda\in\mathbb{Z}^+$
	Note	You can recover work for part (c) in part (b). You cannot recover part (b) work in part (c).
	Note	Candidates are allowed to restart and gain all 2 marks in part (c) from an incorrect part (b).
	Note	Award M1 A1 for the correct answer from no working.

1. (a)	**Alternative methods for part (a)** **Alternative method 1:** Candidates can apply an alternative form of the binomial expansion. $\left\{(4+5x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(5x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(5x)^2$
	B1 $(4)^{\frac{1}{2}}$ or 2
	M1 Any two of three (un-simplified) terms correct.
	A1 All three (un-simplified) terms correct.
	A1 $2+\frac{5}{4}x$ **(simplified fractions)** or allow $2+1.25x$ or $2+1\frac{1}{4}x$
	A1 Accept only $-\frac{25}{64}x^2$ or $-0.390625x^2$
	Note The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(5x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(5x)^2$ without further working is B0M0A0.
	Alternative Method 2: Maclaurin Expansion $\mathrm{f}(x)=(4+5x)^{\frac{1}{2}}$

$\mathrm{f}''(x) = -\frac{25}{4}(4+5x)^{-\frac{3}{2}}$	Correct $\mathrm{f}''(x)$	B1
$\mathrm{f}'(x) = \frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	$\pm a(4+5x)^{-\frac{1}{2}};\ a \neq \pm 1$	M1
	$\frac{1}{2}(4+5x)^{-\frac{1}{2}}(5)$	A1 oe
$\left\{\therefore \mathrm{f}(0)=2,\ \mathrm{f}'(0)=\frac{5}{4} \text{ and } \mathrm{f}''(0)=-\frac{25}{32}\right\}$		
So, $\mathrm{f}(x) = 2+\frac{5}{4}x; -\frac{25}{64}x^2+\ldots$		A1; A1

Question Number	Scheme	Marks
2.	$x^2 - 3xy - 4y^2 + 64 = 0$	
(a)	$\left\{\cancel{\frac{dy}{dx}} = \right\}\quad \underline{2x} - \underline{\underline{\left(3y + 3x\frac{dy}{dx}\right)}} \underline{- 8y\frac{dy}{dx}} = \underline{0}$	M1$\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**	A1 **cso**
		[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} 2x - 3y = 0$	M1
	$y = \frac{2}{3}x$ | $x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$ | $\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$ | $\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\} x = \frac{24}{5}$ or $-\frac{24}{5}$ | $\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\} y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 **cso**
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$ | When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ **cso**	ddM1 A1
		[6] **11**
	Alternative method for part (a)	
(a)	$\left\{\cancel{\frac{dx}{dy}} = \right\}\quad \underline{2x\frac{dx}{dy}} - \underline{\underline{\left(3y\frac{dx}{dy} + 3x\right)}} \underline{- 8y} = \underline{0}$	M1$\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$(2x - 3y)\frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**	A1 **cso**
		[5]

	Question 2 Notes	
2. (a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ from **no working** is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x-3y}{-3x-8y}$ or $\frac{3y-2x}{3x+8y}$ from **no working** is M1A0B1M1A0
	Note	Few candidates will write $2x\,dx - 3y\,dx - 3x\,dy - 8y\,dy = 0$ leading to $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$, o.e. This should get full marks.

2. (a)	**M1**	Differentiates implicitly to include either $\pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-4y^2 \to \pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1	**Both** $x^2 \to \underline{2x}$ **and** $\ldots - 4y^2 + 64 = 0 \to \underline{-8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$
	Note	If an extra term appears then award A0.
	M1	$-3xy \to -3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y$ or $-3x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y$ or $3x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y$ or $3x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y$
	Note	$2x - 3y - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y\frac{\mathrm{d}y}{\mathrm{d}x} \to 2x - 3y = 3x\frac{\mathrm{d}y}{\mathrm{d}x} + 8y\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the "$= 0$" can be implied by the rearrangement of their equation.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}y}{\mathrm{d}x}$. i.e. $\ldots + (-3x - 8y)\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ or $\ldots = (3x + 8y)\frac{\mathrm{d}y}{\mathrm{d}x}$. (Allow combining in 1 variable).
	A1	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	Note	**cso** If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).
2. (b)	**M1**	Sets their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) o.e.
	Note	1st M1 can also be gained by setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero in their "$2x - 3y - 3x\frac{\mathrm{d}y}{\mathrm{d}x} - 8y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$"
	Note	**If their numerator involves one variable only then only the 1st M1 mark is possible in part (b).**
	Note	**If their numerator is a constant then no marks are available in part (b)**
	Note	**If their numerator is in the form** $\pm ax^2 \pm by = 0$ or $\pm ax \pm by^2 = 0$ **then the first 3 marks are possible in part (b).**
	Note	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.
	A1ft	Either • Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ • the follow through result of making either y or x the subject from setting their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero
	dM1	**dependent on the first method mark being awarded.** Substitutes ***either*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) ***by correct solution only***. i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

2. (b) ctd	**ddM1**	**dependent on both previous method marks being awarded in this part.** **<u>Method 1</u>** **Either:** • substitutes their x into their $y = \frac{2}{3}x$ **or** substitutes their y into their $x = \frac{3}{2}y$, **or** • substitutes ***the other of*** their $y = \frac{2}{3}x$ ***or*** their $x = \frac{3}{2}y$ into the original equation, **and achieves either:** • exactly two sets of two coordinates **or** • exactly two distinct values for x and exactly two distinct values for y. **<u>Method 2</u>** **Either:** • substitutes their first x-value, x_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain 1 y-value y_2 **or** • substitutes their first y-value, y_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$, only **by cso.** Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ for 3rd A1.
	Note	$x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ **and** $\left(-\frac{16}{5}, -\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ **and** $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

Question Number	Scheme		Marks
3.	$y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$		
(a)	$\left\{y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$		
	$e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$	M1
		$4\ln 2$ **cao** (Ignore $x = 0$)	A1
			[2]
(b)	$\left\{\int xe^{\frac{1}{2}x}\,dx\right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$	$\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\},\ \alpha > 0, \beta > 0$	M1
		$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$, with or without dx	A1 **(M1 on ePEN)**
	$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\ \{+c\}$	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,dx\right\} = 2x^2$	$4x \to 2x^2$ or $\dfrac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x - xe^{\frac{1}{2}x})\,dx\right\} = \left[2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$		
	$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}\right)$	**See notes**	M1
	$= \left(32(\ln 2)^2 - 32(\ln 2) + 16\right) - (4)$ $= 32(\ln 2)^2 - 32(\ln 2) + 12$	$32(\ln 2)^2 - 32(\ln 2) + 12$, **see notes**	A1
			[3] 8

Question 3 Notes

3. (a)	**M1**	Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
	A1	$4\ln 2$ **cao** stated in part (a) only (Ignore $x = 0$)
(b)	**NOTE**	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha > 0, \beta > 0$. (must be in this form) with or without dx
	A1	$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$ or equivalent, with or without dx. **Can be un-simplified.**
	A1	$2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$ or equivalent with or without $+c$. **Can be un-simplified.**
	Note	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	<u>**SPECIAL CASE:**</u> A candidate who uses $u = x$, $\dfrac{dv}{dx} = e^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

3. (c)	**B1**	$4x \to 2x^2$ or $\frac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	5.19378… without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	5.19378… following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.
	Note	5.19378… from no working is M0A0.

Question Number	Scheme		Marks
4.	$l_1 : \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$. Let $\theta =$ acute angle between l_1 and l_2. **Note: You can mark parts (a) and (b) together.**		
(a)	$\{l_1 = l_2 \Rightarrow \mathbf{i}:\}\ 5 = 8 + 3\mu \Rightarrow \mu = -1$	Finds μ and substitutes their μ into l_2	M1
	So, $\{\overrightarrow{OA}\} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$	$5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$	A1
			[2]
(b)	$\{\mathbf{j}:\ -3 + \lambda = 5 + 4\mu \Rightarrow\}\ -3 + \lambda = 5 + 4(-1) \Rightarrow \lambda = 4$	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$	M1
	$\mathbf{k}:\ p - 3\lambda = -2 - 5\mu \Rightarrow$ $p - 3(4) = -2 - 5(-1) \Rightarrow \underline{p = 15}$ or $\mathbf{k}:\ p - 3\lambda = 3 \Rightarrow$ $p - 3(4) = 3 \Rightarrow \underline{p = 15}$	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda =$ the **k** value of A found in part (a)", substitutes their λ and solves to give $p = ...$	M1
		$p = 15$	A1
			[3]
(c)	$\mathbf{d_1} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix},\ \mathbf{d}_2 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm K\left(\dfrac{0(3) + (1)(4) + (-3)(-5)}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}\right)$	An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	dM1 **(A1 on ePEN)**
	$\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82\ (2\text{ dp})$	anything that rounds to 31.82	A1
			[3]
(d)	$\overrightarrow{OB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix};\ \overrightarrow{AB} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -10 \end{pmatrix}$ $\lvert\overrightarrow{AB}\rvert = \sqrt{6^2 + 8^2 + (-10)^2}\ \{= 10\sqrt{2}\}$	**See notes**	M1
	$\dfrac{d}{10\sqrt{2}} = \sin\theta$	Writes down a correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe.	dM1
	$\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\}\ d = 7.456540753... = 7.46\ (3\text{sf})$	anything that rounds to 7.46	A1
			[3] **11**

4. (b)	**Alternative method for part (b)**		
	$\left\{\begin{array}{l} 3\times \mathbf{j}: \ -9+3\lambda = 15+12\mu \\ \quad \mathbf{k}: \ \ p-3\lambda = -2+5\mu \end{array}\right\} \quad p-9=13+7\mu$	Eliminates λ to write down an equation in p and μ	M1
	$p-9=13+7(-1) \Rightarrow \underline{p=15}$	Substitutes their μ and solves to give $p = ...$	M1
		$p=15$	A1
4. (d)	**Alternative Methods for part (d)** Let X be the foot of the perpendicular from B onto l_1		
	$\mathbf{d}_1 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OX} = \begin{pmatrix} 5 \\ -3 \\ 15 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix}$		
	$\overrightarrow{BX} = \begin{pmatrix} 5 \\ -3+\lambda \\ 15-3\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix}$		
	Method 1		
	$\overrightarrow{BX} \bullet \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -6 \\ -12+\lambda \\ 22-3\lambda \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -12+\lambda-66+9\lambda = 0$ leading to $10\lambda - 78 = 0 \Rightarrow \lambda = \frac{39}{5}$	(Allow a sign slip in copying $\mathbf{d}_1$) Applies $\overrightarrow{BX} \bullet \mathbf{d}_1 = 0$ and solves the resulting equation to find a value for λ.	M1
	$\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12+\frac{39}{5} \\ 22-3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$	Substitutes their value of λ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753...$	awrt 7.46	A1
	Method 2		
	Let $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9\lambda^2$ $= 10\lambda^2 - 156\lambda + 664$ So $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}$	Finds $\beta = \left\lvert\overrightarrow{BX}\right\rvert^2$ in terms of λ, finds $\frac{\mathrm{d}\beta}{\mathrm{d}\lambda}$ and sets this result equal to 0 and finds a value for λ.	M1
	$\left\lvert\overrightarrow{BX}\right\rvert^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$	Substitutes their value of λ into their $\left\lvert\overrightarrow{BX}\right\rvert^2$. **Note:** This mark is dependent upon the previous M1 mark .	dM1
	$d = BX = \sqrt{\frac{278}{5}} = 7.456540753...$	awrt 7.46	A1

		Question 4 Notes
4. (a)	**M1**	Finds μ and substitutes their μ into l_2
	A1	Point of intersection of $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$. Allow $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$.
	Note	You cannot recover the answer for part (a) in part (c) or part (d).
(b)	**M1**	Equates **j** components, substitutes their μ and solves to give $\lambda = ...$
	M1	Equates **k** components, substitutes their λ and their μ and solves to give $p = ...$ **or** equates **k** components to give their "$p-3\lambda =$ the **k** value of A" found in part (b).
	A1	$p = 15$
(c)	**NOTE**	**Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.**
	M1	Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	Note	Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.
	A1	anything that rounds to 31.82. This can also be achieved by $180 - 148.1796... =$ awrt 31.82
	Note	$\theta = 0.5553...^c$ is A0.
	Note	**M1A1 for** $\cos\theta = \left(\dfrac{0-16-60}{\sqrt{(0)^2+(4)^2+(-12)^2}\,.\sqrt{(-3)^2+(-4)^2+(5)^2}}\right) = \dfrac{-76}{\sqrt{160}.\sqrt{50}}$

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1 \times \mathbf{d}_2 = \underline{\begin{pmatrix}0\\1\\-3\end{pmatrix} \times \begin{pmatrix}3\\4\\-5\end{pmatrix}} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\0 & 1 & -3\\3 & 4 & -5\end{vmatrix} = 7\mathbf{i}-9\mathbf{j}-3\mathbf{k}\right\}$	Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$.	<u>M1</u>
$\sin\theta = \dfrac{\sqrt{(7)^2+(-9)^2+(3)^2}}{\sqrt{(0)^2+(1)^2+(-3)^2}\,.\sqrt{(3)^2+(4)^2+(-5)^2}}$	An attempt to apply the vector cross product formula	dM1 **(A1 on ePEN)**
$\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp)	anything that rounds to 31.82	A1

(d)	**M1**	Full method for finding B **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$.
	dM1	**dependent on the first method mark being awarded.** Writes down correct trigonometric equation involving the shortest distance, d. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where "their AB" is a value. and $\theta =$ "their θ" or stated as θ
	A1	anything that rounds to 7.46

Question Number	Scheme		Marks
5.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 **o.e.**
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2,\ x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = $ their "$x^2 + ax + b$", $g(x) = x - 3$	M1
	$\{\text{When } t = 2,\ x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow\ y = \dfrac{x^2 + 2x - 5}{x-3}$, $\{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3] **6**

Question Number	Scheme	Marks
5. (b)	**<u>Alternative Method 1 of Equating Coefficients</u>**	
	$y = \dfrac{x^2+ax+b}{x-3} \Rightarrow y(x-3) = x^2+ax+b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2+32t+10$	
	$x^2+ax+b = (4t+3)^2 + a(4t+3) + b$	
	$(4t+3)^2 + a(4t+3) + b = 16t^2+32t+10$ — Correct method of obtaining an equation in only t, a and b	M1
	t: $24+4a=32 \Rightarrow a=2$ constant: $9+3a+b=10 \Rightarrow b=-5$ — Equates their coefficients in t and finds both $a = \ldots$ and $b = \ldots$	dM1
	$a=2$ and $b=-5$	A1
		[3]
5. (b)	**<u>Alternative Method 2 of Equating Coefficients</u>**	
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$ — Eliminates t to achieve an equation in only x and y	M1
	$y = x-3+8+\dfrac{10}{x-3} \Rightarrow y = x+5+\dfrac{10}{(x-3)}$ $\underline{\underline{y(x-3)}} = (x+5)(x-3)+10 \Rightarrow x^2+ax+b = \underline{\underline{(x+5)(x-3)+10}}$	dM1
	$\Rightarrow y = \dfrac{x^2+2x-5}{x-3}$ **or** equating coefficients to give $a=2$ and $b=-5$ — Correct algebra leading to $y = \dfrac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$	A1 **cso**
		[3]

		Question 5 Notes
5. (a)	**B1**	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 - \frac{5}{2}t^{-2}$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{8t^2 - 5}{2t^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{\mathrm{d}y}{\mathrm{d}t}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by a candidate's $\frac{\mathrm{d}x}{\mathrm{d}t}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}t}$ multiplied by a candidate's $\frac{\mathrm{d}t}{\mathrm{d}x}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{\mathrm{d}y}{\mathrm{d}t}$ and their $\frac{\mathrm{d}x}{\mathrm{d}t}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	**M1**	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\ldots + 8(x-3) + \ldots$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\ldots + 5(x-3) + \ldots$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Question Number	Scheme	Marks
6. (a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x\ ,\ \ x = 1 + 2\sin\theta$	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ — $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{ \int \sqrt{(3-x)(x+1)}\ \mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\ \mathrm{d}x \right\}$	
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$ — Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$	
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ \ 2\cos\theta\ \{\mathrm{d}\theta\}$ — Applies $\cos^2\theta = 1 - \sin^2\theta$ **see notes**	M1
	$= 4\int \cos^2\theta\ \mathrm{d}\theta,\ \ \{k = 4\}$ — $4\int \cos^2\theta \mathrm{d}\theta$ or $\int 4\cos^2\theta \mathrm{d}\theta$ **Note:** $\mathrm{d}\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$ — See notes	B1
		[5]
(b)	$\left\{ k\int \cos^2\theta\ \{\mathrm{d}\theta\} \right\} = \{k\}\int \left(\frac{1+\cos 2\theta}{2}\right) \{\mathrm{d}\theta\}$ — Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$ — Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0,\ \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 **(A1 on ePEN)**
	$\left\{ \text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$	
	$\left\{ = (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ — $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ **or** $\frac{1}{6}\left(8\pi + 3\sqrt{3}\right)$	A1 **cao cso**
		[3] **8**

		Question 6 Notes
6. (a)	**B1**	$\dfrac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta\, d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged}\dfrac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\, dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}\, dx$ is equal to $4\int \cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\dfrac{1}{2}$ which then leads to $\theta = -\dfrac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \dfrac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\dfrac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\dfrac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\dfrac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\dfrac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\dfrac{\pi}{6};\ x = 3,\ \theta = \dfrac{\pi}{2}$
(b)	**NOTE**	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\dfrac{1+\cos 2\theta}{2}\right)$ and ***applies*** it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A ***correct solution in part (b)*** leading to a "two term" exact answer. Eg: $\dfrac{4\pi}{3} + \dfrac{\sqrt{3}}{2}$ **or** $\dfrac{8\pi}{6} + \dfrac{\sqrt{3}}{2}$ **or** $\dfrac{1}{6}\left(8\pi + 3\sqrt{3}\right)$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$		
	$2 \equiv A(P-2) + BP$	Can be implied.	M1
	$A = -1,\ B = 1$	Either one.	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	See notes. **cao, aef**	A1
			[3]
(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{1}{2}P(P-2)\cos 2t$		
	$\displaystyle\int \frac{2}{P(P-2)}\,\mathrm{d}P = \int \cos 2t\ \mathrm{d}t$	can be implied by later working	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	$\pm\lambda\ln(P-2) \pm \mu\ln P$, $\lambda \neq 0,\ \mu \neq 0$	M1
		$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t$	A1
	$\{t = 0,\ P = 3 \Rightarrow\}\ \ \ln 1 - \ln 3 = 0 + c\ \ \ \{\Rightarrow\ c = -\ln 3\ \text{ or }\ \ln(\tfrac{1}{3})\}$	See notes	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$ $\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$		
	$\dfrac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	M1
	$3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t}$ gives $3P - P\mathrm{e}^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$	A complete method of rearranging to make P the subject. **Must have a constant of integration that need not be evaluated (see note)**	dM1
	$P = \dfrac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	A1 * cso
			[7]
(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	States $P = 4$ or applies $P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, $\lambda \neq 0,\ k > 0$ where λ and k are numerical values and λ can be 1	M1
	$t = 0.4728700467...$	anything that rounds to 0.473 Do not apply isw here	A1
			[3] **13**

Question Number	Scheme		Marks
	Method 2 for Q7(b)		
7. (b)	$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t \;\; (+c)$	As before for…	**B1M1A1**
	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$		
	$\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c}$ or $\frac{(P-2)}{P} = Ae^{\frac{1}{2}\sin 2t}$	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c$, $\lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **Must have a constant of integration that need not be evaluated (see note)**	**3rd M1**
	$(P-2) = APe^{\frac{1}{2}\sin 2t} \Rightarrow P - APe^{\frac{1}{2}\sin 2t} = 2$ $\Rightarrow P(1 - Ae^{\frac{1}{2}\sin 2t}) = 2 \Rightarrow P = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2t})}$	A complete method of rearranging to make P the subject. Condone sign slips or constant errors. **Must have a constant of integration that need not be evaluated (see note)**	**4th dM1**
	$\{t = 0, P = 3 \Rightarrow\} \quad 3 = \frac{2}{(1 - Ae^{\frac{1}{2}\sin 2(0)})}$	See notes **(Allocate this mark as the 2nd M1 mark on ePEN).**	**2nd M1**
	$\left\{\Rightarrow 3 = \frac{2}{(1-A)} \Rightarrow A = \frac{1}{3}\right\}$		
	$\Rightarrow P = \frac{2}{\left(1 - \frac{1}{3}e^{\frac{1}{2}\sin 2t}\right)} \Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ *	Correct proof.	**A1 * cso**

Question 7 Notes

7. (a)	**M1**	Forming a correct identity. For example, $2 \equiv A(P-2) + BP$ from $\frac{2}{P(P-2)} = \frac{A}{P} + \frac{B}{(P-2)}$
	Note	A and B are not referred to in question.
	A1	Either one of $A = -1$ or $B = 1$.
	A1	$\frac{1}{(P-2)} - \frac{1}{P}$ or any equivalent form. This answer ***cannot*** be recovered from part (b).
	Note	M1A1A1 can also be given for a candidate who finds both $A = -1$ and $B = 1$ and $\frac{A}{P} + \frac{B}{(P-2)}$ is seen in their working.
	Note	Candidates can use 'cover-up' rule to write down $\frac{1}{(P-2)} - \frac{1}{P}$, so as to gain all three marks.
	Note	Equating coefficients from $2 \equiv A(P-2) + BP$ gives $A + B = 2, -2A = 2 \Rightarrow A = -1, \; B = 1$

7. (b)	**B1** **Note**	Separates variables as shown on the Mark Scheme. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **Eg:** $\int \frac{2}{P^2-2P}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$ or $\int \frac{1}{P(P-2)}\,\mathrm{d}P = \frac{1}{2}\int \cos 2t\,\mathrm{d}t$ **o.e.** are also fine for B1.
	1st M1 **Note**	$\pm\lambda\ln(P-2) \pm \mu\ln P,\ \lambda \neq 0,\ \mu \neq 0$. Also allow $\pm\lambda\ln(M(P-2)) \pm \mu\ln NP$; M, N can be 1. Condone $2\ln(P-2) + 2\ln P$ or $2\ln(P(P-2))$ or $2\ln(P^2-2P)$ or $\ln(P^2-2P)$
	1st A1 **2nd M1**	Correct result of $\ln(P-2) - \ln P = \frac{1}{2}\sin 2t$ or $2\ln(P-2) - 2\ln P = \sin 2t$ o.e. with or without $+c$ Some evidence of using both $t = 0$ and $P = 3$ in an integrated equation containing a constant of integration. Eg: c or A, etc.
	3rd M1 **4th M1** **Note**	Starting from an equation of the form $\pm\lambda\ln(P-\beta) \pm \mu\ln P = \pm K\sin\delta t + c,\ \lambda, \mu, \beta, K, \delta \neq 0$, applies a fully correct method to eliminate their logarithms. **dependent on the third method mark being awarded.** A complete method of rearranging to make P the subject. Condone sign slips or constant errors. For the 3rd M1 and 4th M1 marks, a candidate needs to have included a constant of integration, in their working. eg. $c, A, \ln A$ or an evaluated constant of integration.
	2nd A1	Correct proof of $P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$. **Note:** This answer is given in the question.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c$ followed by $\frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is 3rd M0, 4th M0, 2nd A0.
	Note	$\ln\left(\frac{(P-2)}{P}\right) = \frac{1}{2}\sin 2t + c \to \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t + c} \to \frac{(P-2)}{P} = e^{\frac{1}{2}\sin 2t} + e^c$ is final M1M0A0
	<u>**4th M1 for making *P* the subject**</u> **Note there are three type of manipulations here which are considered acceptable for making *P* the subject.** **(1)** M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3(P-2) = Pe^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = Pe^{\frac{1}{2}\sin 2t} \Rightarrow P(3 - e^{\frac{1}{2}\sin 2t}) = 6$ $\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ **(2)** M1 for $\frac{3(P-2)}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - \frac{6}{P} = e^{\frac{1}{2}\sin 2t} \Rightarrow 3 - e^{\frac{1}{2}\sin 2t} = \frac{6}{P} \Rightarrow\Rightarrow P = \frac{6}{(3 - e^{\frac{1}{2}\sin 2t})}$ **(3)** M1 for $\left\{\ln(P-2) + \ln P = \frac{1}{2}\sin 2t + \ln 3 \Rightarrow\right\} P(P-2) = 3e^{\frac{1}{2}\sin 2t} \Rightarrow P^2 - 2P = 3e^{\frac{1}{2}\sin 2t}$ $\Rightarrow (P-1)^2 - 1 = 3e^{\frac{1}{2}\sin 2t}$ leading to $P = ..$	
(c)	**M1** **M1** **A1**	States $P = 4$ or applies $P = 4$ Obtains $\pm\lambda\sin 2t = \ln k$ or $\pm\lambda\sin t = \ln k$, where λ and k are numerical values and λ can be 1 anything that rounds to 0.473. ***(Do not apply isw here)***
	Note	***Do not apply ignore subsequent working for A1.*** (Eg: 0.473 followed by 473 years is A0.)
	Note	<u>Use of $P = 4000$</u>: Without the mention of $P = 4$, $\frac{1}{2}\sin 2t = \ln 2.9985$ or $\sin 2t = 2\ln 2.9985$ or $\sin 2t = 2.1912...$ will usually imply M0M1A0
	Note	<u>Use of Degrees:</u> $t =$ awrt 27.1 will usually imply M1M1A0

Question Number	Scheme	Marks
8. (a)	$\left\{y = 3^x \Rightarrow\right\}\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3^x \ln 3$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3^x \ln 3$ or $\ln 3\left(\mathrm{e}^{x\ln 3}\right)$ or $y\ln 3$	B1
	Either T: $y - 9 = 3^2 \ln 3(x-2)$ **or T:** $y = (3^2\ln 3)x + 9 - 18\ln 3$, where $9 = (3^2\ln 3)(2) + c$ See notes	M1
	{Cuts x-axis $\Rightarrow y = 0 \Rightarrow$}	
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2\ln 3)x + 9 - 18\ln 3$, Sets $y = 0$ in their tangent equation and progresses to $x = ...$	M1
	So, $x = 2 - \dfrac{1}{\ln 3}$ $2 - \dfrac{1}{\ln 3}$ or $\dfrac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 **cso**
		[4]
(b)	$V = \pi\int\left(3^x\right)^2\{\mathrm{d}x\}$ or $\pi\int 3^{2x}\{\mathrm{d}x\}$ or $\pi\int 9^x\{\mathrm{d}x\}$ $V = \pi\int\left(3^x\right)^2$ with or without $\mathrm{d}x$, which can be implied	B1 o.e.
	$= \{\pi\}\left(\dfrac{3^{2x}}{2\ln 3}\right)$ or $= \{\pi\}\left(\dfrac{9^x}{\ln 9}\right)$ Eg: either $3^{2x} \to \dfrac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \dfrac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$, $\underline{\underline{\alpha \in}}$	M1
	$3^{2x} \to \dfrac{3^{2x}}{2\ln 3}$ or $9^x \to \dfrac{9^x}{\ln 9}$ or $\mathrm{e}^{2x\ln 3} \to \dfrac{1}{2\ln 3}\left(\mathrm{e}^{2x\ln 3}\right)$	A1 o.e.
	$\left\{V = \pi\int_0^2 3^{2x}\,\mathrm{d}x = \{\pi\}\left[\dfrac{3^{2x}}{2\ln 3}\right]_0^2\right\} = \{\pi\}\left(\dfrac{3^4}{2\ln 3} - \dfrac{1}{2\ln 3}\right)\ \left\{= \dfrac{40\pi}{\ln 3}\right\}$ **Dependent on the previous method mark.** Substitutes $x = 2$ and $x = 0$ and subtracts the correct way round.	dM1
	$V_{\text{cone}} = \dfrac{1}{3}\pi(9)^2\left(\dfrac{1}{\ln 3}\right)\ \left\{= \dfrac{27\pi}{\ln 3}\right\}$ $V_{\text{cone}} = \dfrac{1}{3}\pi(9)^2\left(2 - \text{their } (a)\right)$. See notes.	B1ft
	$\left\{\mathrm{Vol}(S) = \dfrac{40\pi}{\ln 3} - \dfrac{27\pi}{\ln 3}\right\} = \underline{\dfrac{13\pi}{\ln 3}}$ $\dfrac{13\pi}{\ln 3}$ or $\dfrac{26\pi}{\ln 9}$ or $\dfrac{26\pi}{2\ln 3}$ etc., isw	A1 o.e.
	$\left\{\text{Eg: } p = 13\pi,\ q = \ln 3\right\}$	**[6]** **10**
(b)	<u>**Alternative Method 1: Use of a substitution**</u>	
	$V = \pi\int\left(3^x\right)^2\{\mathrm{d}x\}$	B1 o.e.
	$\left\{u = 3^x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3^x\ln 3 = u\ln 3\right\}$ $V = \{\pi\}\int\dfrac{u^2}{u\ln 3}\{\mathrm{d}u\} = \{\pi\}\int\dfrac{u}{\ln 3}\{\mathrm{d}u\}$	
	$= \{\pi\}\left(\dfrac{u^2}{2\ln 3}\right)$ $\left(3^x\right)^2 \to \dfrac{u^2}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)u^2$, where $u = 3^x$	M1
	$\left(3^x\right)^2 \to \dfrac{u^2}{2(\ln 3)}$, where $u = 3^x$	A1
	$\left\{V = \pi\int_0^2\left(3^x\right)^2\mathrm{d}x = \{\pi\}\left[\dfrac{u^2}{2\ln 3}\right]_1^9\right\} = \{\pi\}\left(\dfrac{9^2}{2\ln 3} - \dfrac{1}{2\ln 3}\right)\ \left\{= \dfrac{40\pi}{\ln 3}\right\}$ Substitutes limits of 9 and 1 in u (or 2 and 0 in x) and subtracts the correct way round.	dM1
	then apply the main scheme.	

		Question 8 Notes
8. (a)	**B1**	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y \ln 3$. Can be implied by later working.
	M1	Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{dy}{dx}$ which is a function of *x* or *y* to find m_T **and** • **either** applies $y - 9 = \left(\text{their } m_T\right)\left(x - 2\right)$, where m_T is a numerical value. • **or** applies $y = \left(\text{their } m_T\right)x + \text{their } c$, where m_T is a numerical value and *c* is found by solving $9 = \left(\text{their } m_T\right)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their ***tangent*** equation, where m_T is a numerical value, (seen or implied) and progresses to $x = ...$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$, where λ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$) is M0 M0 in part (a).
	Note	Candidates who invent a value for m_T (which bears no resemblance to their gradient function) **cannot** gain the 1st M1 and 2nd M1 mark in part (a).
	Note	A decimal answer of 1.089760773… (without a correct **exact** answer) is A0.
8. (b)	**B1**	A correct expression for the volume with or without dx
	Note	**Eg:** Allow B1 for $\pi\int\left(3^x\right)^2\{dx\}$ or $\pi\int 3^{2x}\{dx\}$ or $\pi\int 9^x\{dx\}$ or $\pi\int\left(e^{x\ln 3}\right)^2\{dx\}$ or $\pi\int\left(e^{2x\ln 3}\right)\{dx\}$ or $\pi\int e^{x\ln 9}\{dx\}$ with or without dx
	M1	Either $3^{2x} \to \frac{3^{2x}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)3^{2x}$ or $9^x \to \frac{9^x}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^x$ $e^{2x\ln 3} \to \frac{e^{2x\ln 3}}{\pm\alpha(\ln 3)}$ or $\pm\alpha(\ln 3)e^{2x\ln 3}$ or $e^{x\ln 9} \to \frac{e^{x\ln 9}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)e^{x\ln 9}$, etc where $\alpha \in$
	Note	$3^{2x} \to \frac{3^{2x+1}}{\pm\alpha(\ln 3)}$ or $9^x \to \frac{9^{x+1}}{\pm\alpha(\ln 3)}$ are allowed for M1
	Note	$3^{2x} \to \frac{3^{2x+1}}{2x+1}$ or $9^x \to \frac{9^{x+1}}{x+1}$ are both M0
	Note	M1 can be given for $9^{2x} \to \frac{9^{2x}}{\pm\alpha(\ln 9)}$ or $\pm\alpha(\ln 9)9^{2x}$
	A1	Correct integration of 3^{2x}. Eg: $3^{2x} \to \frac{3^{2x}}{2\ln 3}$ or $\frac{3^{2x}}{\ln 9}$ or $9^x \to \frac{9^x}{\ln 9}$ or $e^{2x\ln 3} \to \frac{1}{2\ln 3}\left(e^{2x\ln 3}\right)$
	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.

	dM1	**dependent on the previous method mark being awarded.** Attempts to apply $x = 2$ and $x = 0$ to integrated expression and subtracts the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	B1ft	$V_{\text{cone}} = \frac{1}{3}\pi(9)^2\left(2 - \text{their answer to part }(a)\right)$. Sight of $\frac{27\pi}{\ln 3}$ implies the B1 mark.
	Note	Alternatively they can apply the volume formula to the line segment. They need to achieve the result highlighted by **** on either page 29 or page 30 in order to obtain the B1ft mark.
	A1	$\frac{13\pi}{\ln 3}$ or $\frac{26\pi}{\ln 9}$ or $\frac{26\pi}{2\ln 3}$, etc. , where their answer is in the form $\frac{p}{q}$
	Note	The π in the volume formula is only needed for the 1st B1 mark and the final A1 mark.
	Note	A decimal answer of 37.17481128… (without a correct **exact** answer) is A0.
	Note	A candidate who applies $\int 3^x \mathrm{d}x$ will either get B0 M0 A0 M0 **B0** A0 **or** B0 M0 A0 M0 **B1** A0
	Note	$\pi\int 3^{x^2}\,\mathrm{d}x$ unless recovered is B0.
	Note	**<u>Be careful! A correct answer may follow from incorrect working</u>** $V = \pi\int_0^2 3^{x^2}\,\mathrm{d}x - \frac{1}{3}\pi(9)^2\left(\frac{1}{\ln 3}\right) = \pi\left[\frac{3^{x^2}}{2\ln 3}\right]_0^2 - \frac{27\pi}{\ln 3} = \frac{\pi 3^4}{2\ln 3} - \frac{\pi}{2\ln 3} - \frac{27\pi}{\ln 3} = \frac{13\pi}{\ln 3}$ would score B0 M0 A0 dM0 M1 A0.
8. (b)	**<u>2nd B1ft mark for finding the Volume of a Cone</u>**	

$$V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2}\left(9x\ln 3 - 18\ln 3 + 9\right)^2 \mathrm{d}x$$

$$= \pi\left[\frac{\left(9x\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right]_{2-\frac{1}{\ln 3}\text{ or their part (a) answer}}^{2} \quad ****$$

Award B1ft here where their lower limit is $2 - \frac{1}{\ln 3}$ or their part (a) answer.

$$= \pi\left(\left(\frac{\left(18\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right) - \left(\frac{\left(9\left(2 - \frac{1}{\ln 3}\right)\ln 3 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \pi\left(\left(\frac{729}{27\ln 3}\right) - \left(\frac{\left(18\ln 3 - 9 - 18\ln 3 + 9\right)^3}{27\ln 3}\right)\right)$$

$$= \frac{27\pi}{\ln 3}$$

8. (b)	**2nd B1ft mark for finding the Volume of a Cone** **Alternative method 2:** $V_{\text{cone}} = \pi\int_{2-\frac{1}{\ln 3}}^{2} (9x\ln 3 - 18\ln 3 + 9)^2\,\mathrm{d}x$ $= \pi\int_{2-\frac{1}{\ln 3}}^{2} \left(81x^2(\ln 3)^2 - 324x(\ln 3)^2 + 162x\ln 3 - 324\ln 3 + 324(\ln 3)^2 + 81\right)\mathrm{d}x$ $= \pi\left[27x^3(\ln 3)^2 - 162x^2(\ln 3)^2 + 81x^2\ln 3 - 324x\ln 3 + 324x(\ln 3)^2 + 81x\right]_{2-\frac{1}{\ln 3}}^{2}$ Award B1ft here where their lower limit is $2 - \dfrac{1}{\ln 3}$ or their part (a) answer. **** $= \pi\left(\left(216(\ln 3)^2 - 648(\ln 3)^2 + 324\ln 3 - 648\ln 3 + 648(\ln 3)^2 + 162\right) - \left(27\left(2-\frac{1}{\ln 3}\right)^3(\ln 3)^2 - 162\left(2-\frac{1}{\ln 3}\right)^2(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)^2\ln 3 - 324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(27\left(8 - \frac{12}{\ln 3} + \frac{6}{(\ln 3)^2} - \frac{1}{(\ln 3)^3}\right)(\ln 3)^2 - 162\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)(\ln 3)^2 + 81\left(4 - \frac{4}{\ln 3} + \frac{1}{(\ln 3)^2}\right)\ln 3 - 324\left(2-\frac{1}{\ln 3}\right)\ln 3 + 324\left(2-\frac{1}{\ln 3}\right)(\ln 3)^2 + 81\left(2-\frac{1}{\ln 3}\right)\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3} - 648(\ln 3)^2 + 648\ln 3 - 162 + 324\ln 3 - 324 + \frac{81}{\ln 3} - 648\ln 3 + 324 + 648(\ln 3)^2 - 324\ln 3 + 162 - \frac{81}{\ln 3}\right)\right)$ $= \pi\left(\left(216(\ln 3)^2 - 324\ln 3 + 162\right) - \left(216(\ln 3)^2 - 324\ln 3 + 162 - \frac{27}{\ln 3}\right)\right)$ $= \dfrac{27\pi}{\ln 3}$

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom